Convex Segmentation Model for Images with Intensity Inhomogeneity

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***Abstract*—Intensity inhomogeneity is a challenging problem in the field of image processing. Many variational models have been developed for this purpose up to now such as Chen Vese, Li-Kim (LK)and Wu-He (WH) models. In this paper we proposed a newly segmentation model which will be capable for images with intensity inhomogeneity. Furthermore the performance of the proposed model compare to exiting models as shown in the experimental section.**

***Keywords*—Image segmentation, Level set, Functional minimization, Numerical method.**

I. Introduction

Image segmentation is one of the most important and complicated task in the field of image processing and computer vision. There are different techniques developed for this task such as histogram analysis and thresholding [?], [?], [?], region growing [?], [?], edge detection and active contours [?], [?], [?]. Active contour models are widely used in image segmentation due to their robustness and reliability. These models are formulated as energy minimization problems and can be categorized broadly into edge-based models [?], [?], [?], [?], and region-based models [?], [?], [?], [?].

Various models for image segmentation have been extensively studied and successfully implemented in image analysis, pattern recognition, image understanding, computer vision, etc. There are two different segmentation classes: 1) global segmentation, where the contour of all the objects in a given image is required to be segmented, and 2) interactive selective segmentation where the task of segmentation is to segment a particular object feature of the given image.

Edge based models employ image gradient information and edge detector functions to attract the dynamic contours toward the boundaries of objects, and the region based ones make use of image intensities (certain homogeneity) to guide the motion of active contours. Compared with the edge-based models, region-based models do not rely on any edge and gradient information and are less sensitive to the noise and clutter. Moreover, the region-based models are usually less dependent on the initialization since they exploit the global region information of the image statistics. Therefore, in this paper, we mainly focus on the region-based models. One of the most popular region-based models is Chan-Vese (C-V) model [?], which, as a special case of Mumford-Shah energy functional[?], is defined by minimizing an energy functional to approximate the image in piecewise constant forms. The energy functional of the model can be written as:

$$F_{CV}(\Gamma, c_1, c_2) = \mu \cdot (length(\Gamma)) + \nu \cdot area(inside(\Gamma)) + \lambda_1 \int_{inside(\Gamma)} |I_0 - d_1|^2 dxdy + \lambda_2 \int_{outside(\Gamma)} |I_0 - d_2|^2 dxdy, \quad (1)$$

In terms of level set formulation, the equation (**??**) becomes:

$$F_{CV}(\phi, d_1, d_2) = \mu \int_\Omega |\nabla H(\phi)| dxdy + \nu \int_\Omega H(\phi) dxdy + \lambda_1 \int_\Omega |I_0 - d_1|^2 H(\phi) dxdy + \lambda_2 \int_\Omega |I_0 - d_2|^2 (1 - H(\phi)) dxdy, \quad (2)$$

Minimizer c_1 and c_2 are define as follow:

$$c_1 = \frac{\int_\Omega I_0^2 (H_c(\phi)}{\int_\Omega I_0) H_c(\phi)}, \qquad c_2 = \frac{\int_\Omega I_0^2 (1 - H_c(\phi(x)))}{\int_\Omega I_0 (1 - H_c(\phi))}. \quad (3)$$

Minimization of the functional (**??**) with respect ϕ leads to the following PDE:

$$\begin{cases} \dfrac{\partial \phi}{\partial t} = \delta_\epsilon(\phi) \Big[\mu \nabla \cdot \Big(\dfrac{\nabla \phi}{|\nabla \phi|} \Big) - \nu - \lambda_1 (I_0 - d_1)^2 + \lambda_2 (I_0 - d_2)^2 \Big], \\ \phi(t, x, y) = \phi_0(x, y), \qquad \text{in } \Omega, \\ \dfrac{\partial \phi}{\partial n} = 0, \qquad \text{on } \partial\Omega. \end{cases} \quad (4)$$

CV model may not segment images having intensity inhomogeneity, however it is only design for homogeneous images. The rest of the paper is organized as follows. Section **??**, reviews some classical models and indicates their limitations. Section **??** describes the variational formulation of our model. Section **??**, presents the experimental results. Section **??** concludes our work with a discussion and future directions.

II. Background

Segmentation of intensity inhomogeneity images is a challenging problem in the field of image processing. Many variational models[?], [?], [?], [?], [?], [?] developed to overcome this problem. But we shall only review two models below that are directly related to this work.

A. The LBF Model

Li et al. [?] proposed a local binary fit (LBF) which deals with intensity inhomogeneity of the images. The model utilizes a kernel function to enhance the CV model [?]. The fitting energy term of the model is given as follows:

$$\begin{aligned} F_{\epsilon}^{LBF}(\phi, f_1, f_2) &= \mu \int_{\Omega} \partial(\phi)|\nabla H|dx \quad (5) \\ +\nu \int_{Omega} (|\nabla\phi| - 1)^2 dx &+ \lambda_1 \int \Big[\int K_{\sigma}(x-y)|I_0(y) \\ -f_1(x)|^2 H(\phi(y))dy\Big]dx &+ \lambda_2 \int \Big[\int K_{\sigma}(x-y)|I_0(y) \\ -f_2(x)|^2(1 - H(\phi(y)))dy\Big]dx \end{aligned}$$

where K_σ is the gaussian kernel having standard deviation σ, u_0 is given image, f_1 and f_2, are the smooth functions which fits the given image locally inside and outside of the contour C respectively. The minimization of functional (**??**) leads to the following equations:

$$f_1(x) = \frac{K_{\sigma}(x)[I_0(x)H_{\epsilon}(\phi(x))]}{K_{\sigma}(x)H_{\epsilon}(\phi(x))} \quad (6)$$

$$f_2(x) = \frac{K_{\sigma}(x)[I_0(x)(1 - H_{\epsilon}(\phi(x)))]}{K_{\sigma}(x)(1 - H_{\epsilon}(\phi(x)))},$$

and

$$\frac{\partial\phi}{\partial t} = -\delta_{\epsilon}(\phi)\Big(\lambda_1 e_1(x) - \lambda_2 e_2(x)\Big) + \nu\delta_{\epsilon}(\phi)div\Big(\frac{\nabla\phi}{|\nabla_{\phi}|}\Big) \quad (7)$$
$$+\mu\Big(\nabla^2\phi - div\Big(\frac{\nabla\phi}{|\nabla\phi|}\Big)\Big),$$

where

$$e_1(x) = \int_{\Omega} K_{\sigma}(y-x)|I_0(x) - f_1(y)|^2,\ e_2(x) \quad (8)$$
$$= \int_{\Omega} K_{\sigma}(y-x)|I_0(x) - f_2(y)|^2 dy.$$

The LBF model works well as compared to the CV model in those images having intensity inhomogeneity. However, it may not work properly in multi-region segmentation.

B. Wu-He (WH) model

To improve the CV model for images with intensity inhomogeneity, Wu and He [?] proposed a coefficient of variation based model (CVM) with a strictly convex energy functional in a level set formulation of the form:

$$F^{WH}(\phi, c_1, c_2) = \lambda \int_{\Omega} \frac{(I_0 - c_1)^2}{c_1^2}(\phi + 1)^2 dx,$$
$$+ \int_{\Omega} \frac{(I_0 - c_2)^2}{c_2^2}(\phi - 1)^2 dx, \quad (9)$$

where $\lambda > 0$. Minimizer c_1 and c_2 are define as follow:

$$c_1 = \frac{\int_{\Omega} I_0^2(H_c(\phi)}{\int_{\Omega} I_0)H_c(\phi)}, \qquad c_2 = \frac{\int_{\Omega} I_0^2(1 - H_c(\phi(x)))}{\int_{\Omega} I_0(1 - H_c(\phi))}. \quad (10)$$

Minimization of the functional (**??**) with respect ϕ leads to the following PDE:

$$\phi_t = -\lambda\frac{(I_0 - c_1)^2}{c_1^2}(\phi + 1) - \frac{(I_0) - c_2)^2}{c_2^2}(\phi - 1) \quad (11)$$

The segmentation results of WH model are better than the CV model, however, it is not designed for images with sever intensity inhomogeneity.

III. The Proposed Model

Now we will design a method which is capable to minimize the intensity inhomogeneity and after that we will proposed our model which can tackle images having intensity inhomogeneity. In our proposed model the given image I_0 describe as follows:

$$I_0(x,y) = \zeta(x,y)I(x,y) + F(x,y),$$

where $I(x,y)$ is the homogenous image while $\zeta(x,y)$ is intensity inhomogeneity and $F(x,y)$ is the additive noise. We can approximate noise with the Gaussian distribution if the signal to noise ratio of $I_0(x,y)$ is not too much low. In order to simplify this computation, the noise is ignored and logarithmic transform of the intensity is used as follows:

$$\log I_0(x,y) = \log\zeta(x,y) + \log I(x,y). \quad (12)$$

In order to analyze the image in small regions, first we construct a multi-scale average filter. The single filtered image v_{D_1} is defined as:

$$I_{0_1} = \frac{I_0 * K}{1 * K} \quad (13)$$

Now the dual filtered image I_{0_2} is defined as follows:

$$I_{0_2} = \frac{I_{0_1} * K}{1 * K} \quad (14)$$

Now replace $\zeta(x,y)$ in (**??**) by I_{0_2}, we get the transformation:

$$\log\hat{I}(x,y) = \log I_0(x,y) - \log I_{0_2}(x,y) + \log C_n, \quad (15)$$

where $\hat{I}(x,y)$ is the approximation of $I(x,y)$ which is inhomogeneity-free image, C_n is the normalized constant which preserves the mean intensity of $\hat{I}(x,y)$. We write equation (**??**) for simplicity as:

$$\hat{I}(x,y) = \frac{C_n I_0(x,y)}{I_{0_2}(x,y)} \quad (16)$$

We propose the following functional of minimization:

$$F^{WH}(\phi, c_1, c_2) = \lambda \int_{\Omega} \frac{(\hat{I}(x,y) - c_1)^2}{c_1^2}(\phi + 1)^2 dx \quad (17)$$
$$+ \int_{\Omega} \frac{(\hat{I}(x,y) - c_2)^2}{c_2^2}(\phi - 1)^2 dx,$$

by minimization with respect to c_1 and c_2 we get the following results:

$$c_1 = \frac{\int_\Omega \hat{I}(x,y)^2 (H_c(\phi) dx}{\int_\Omega \hat{I}(x,y)) H_c(\phi) dx}, \tag{18}$$

$$c_2 = \frac{\int_\Omega \hat{I}(x,y)^2 (1 - H_c(\phi(x))) dx}{\int_\Omega \hat{I}(x,y)(1 - H_c(\phi)) dx}.$$

Minimization of the functional (**??**) with respect ϕ leads to the following PDE:

$$\phi_t = -\lambda \frac{(\hat{I}(x,y) - c_1)^2}{c_1^2} (\phi + 1) - \frac{(\hat{I}(x,y)) - c_2)^2}{c_2^2} (\phi - 1) \tag{19}$$

The segmentation results of the proposed model are better than the Wu-He model on those images having sever intensity inhomogeneity.

IV. Experimental Results

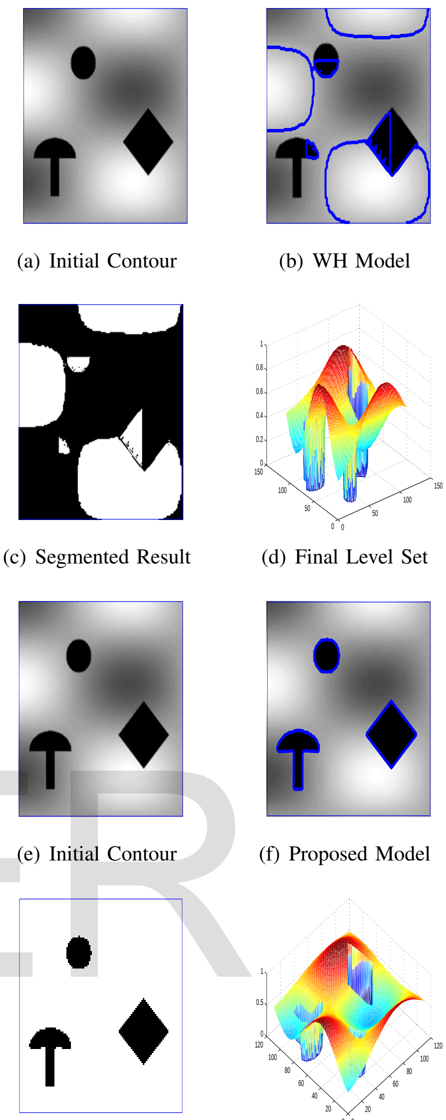
In this section we will show some experimental results of the proposed model and the WH model on the images having intensity.

In Fig.**??** shows the performance of the proposed model on image having background intensity inhomogeneity, the first row show that performance of proposed model, while second row show the result of proposed model. The parameter used for our proposed model as λ = 2, Image Size 200×200 and Iteration= 100.

Similarly, In Fig. **??** and In Fig. **??** segmentation performance of the proposed model on images with intensity inhomogeneity, the first row show that performance of proposed model, while second row show the result of proposed model. The parameter used for our proposed model as λ = 2, Image Size 200×200 and Iteration= 100.

V. Conclusion

In this paper, Convex Segmentation Model for Images with Intensity Inhomogeneity. In Convex Segmentation Model, we set dual filter technique to handle intensity inhomogeneity. The experimental results of the proposed Convex Segmentation Model compared with Wu He model on synthetic images show efficient and robust performance. In future, we plan to extend the proposed algorithm to selective segmentation and video segmentation.



(a) Initial Contour (b) WH Model
(c) Segmented Result (d) Final Level Set
(e) Initial Contour (f) Proposed Model
(g) Segmented Result (h) Final Level Set

Figure 1: Performance of WH and our Proposed model in segmenting images with intensity inhomogeneity. The parameter used for our proposed model as λ = 2, Image Size 200×200 and Iteration= 100.

References

[1] M. S. Khan, N. Badshah, H. Ali, G. Ayub. An Efficient Variational Model for Vector Valued Image Segmentation Student Research Paper Conference, 3(1): 2016.

[2] N. Badshah, K. Chen, H. Ali, G. Murtaza. Coefficient of Variation Based Image Selective Segmentation Model Using Active Contours. East Asian Journal on Applied Mathematics, 2(2): 150–169, 2012.

[3] X. F. Wang, D. S. Huanga, H. Xu. An efficient local ChanVese model for image segmentation. Pattern Recognition, 43: 603–618, 2009.

[4] G. Murtaza, H. Ali and N. Badshah. A Robust Local Model for Segmentation Based on Coefficient of Variation. J. Information & Communication Tech., 5(1): 30-39, 2011.

[5] J. Malik, Th. Leung and J. Shi. Contour and texture analysis for image segmentation. Int. J. Comput. Vision., 43(1): 7-27, 2011.

[6] D. Senand, S. K. Pal. Histogram thresholding using fuzzyand rough measures of association error. IEEE Trans. Image Pro., 18(4): 879-888, 2009.

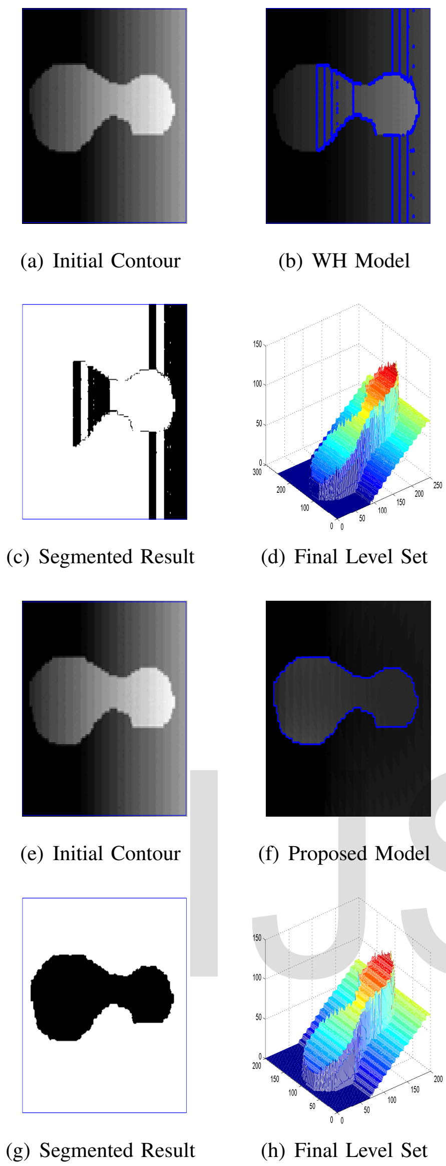
[7] L. Vincent and Soille. Watersheds in Digital Spaces - an efficient algorithm based on immersion. IEEE Trans. Pattern Analysis and Machine Learning, 6: 583–598, 1994.

[8] R. Adams and L. Bischof. Seeded region growing. IEEE T. Pattern Anal., 16(6): 641 647, 1994.

[9] S. W. Zucker. Region growing: childhood and adolescence. Comput. Graph. Image Proc., 5: 382-399, 1976.

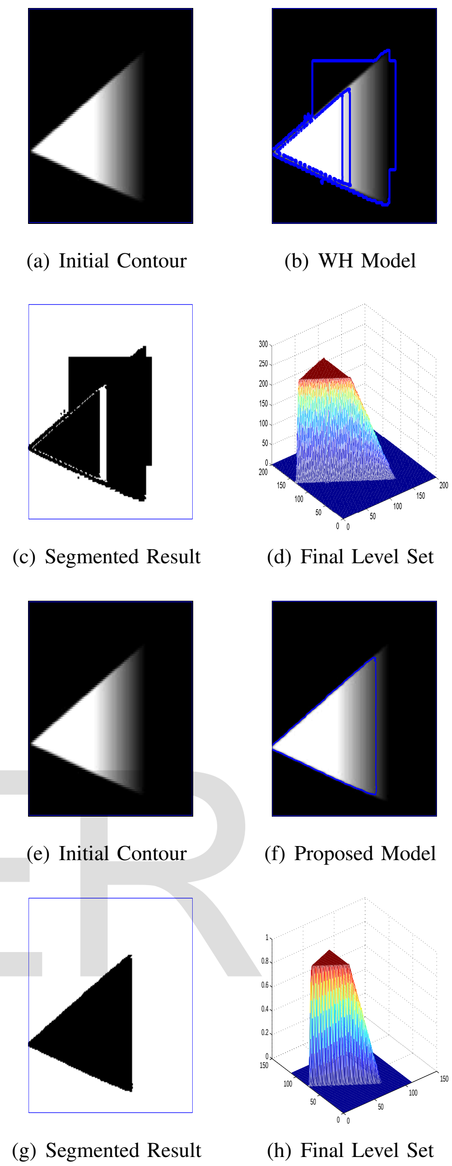
[10] T. F. Chan, J. Shen. Image processing and analysis. Society for Industrial and Applied Mathematics SIAM, PA 19104-2688, 2005.

[11] D. Mumford and J. Shah. Optimal approximation by piecewise smooth functions and associated variational problems. Communications on Pure Applied Mathematics, 42: 577–685, 1989.



(a) Initial Contour (b) WH Model (c) Segmented Result (d) Final Level Set (e) Initial Contour (f) Proposed Model (g) Segmented Result (h) Final Level Set

Figure 2: Performance of WH and our Proposed model in segmenting images with intensity inhomogeneity. The parameter used for our proposed model as $\lambda = 1.5$, Image Size 250×250 and Iteration= 100.



(a) Initial Contour (b) WH Model (c) Segmented Result (d) Final Level Set (e) Initial Contour (f) Proposed Model (g) Segmented Result (h) Final Level Set

Figure 3: Performance of WH and our Proposed model in segmenting images with intensity inhomogeneity. The parameter used for our proposed model as $\lambda = 1$, Image Size 250×250 and Iteration= 100.

[12] T. F. Chan and L. A. Vese. Active contours without edges. IEEE Transactions on Image Processing, 10(2): 266–277, 2001.

[13] V. Caselles, R. Kimmel and G. Sapiro. Geodesic Active Contours. International Jornal of Computer Vision, 22(1): 61–79, 1997.

[14] Y. Wu,C. He. A convex variational level set model for image segmentation. Singal processing. 106:123-133, 2015.

[15] K. Zhang, L. Zhang, H. Song, W. Zhou. Active contours with selective local or global segmentation: A new formulation and level set method. Image and Vision Computing, 28: 668–676, 2010.

[16] S. Osher and R. Fedkiw Γ-Level Set Methods and Dynamic Implicit Surfaces, springer verlag, 2003 Lec. Notes Comp. Sci., 3708: 499–506, 2005.

[17] R. Goldenberg and R. Kimmel and E. Rivlin and M. Rudzsky. Fast geodesic active contours. IEEE Transaction on image processing, 10: 1467–1475, 2001.

[18] N. Paragios and R. Deriche. Geodesic Active Contours and Level Sets for the Detection and Tracking of Moving Objects IEEE Transactions on Pattern Analysis and Machine Intelligence, 22: 266–280, 2000.

[19] M. Rudzsky, E. Rivlin, R. Kimmel and R. Goldenberg. Fast geodesic active contours. Scale-Space Theories in Computer Vision, 1682: 34–45, 1999.

[20] G. Aubert, P. Kornprobst. Mathematical problems in image processing: Partial Differential Equations and the Calculus of Variations. Springer, 2002.

[21] J. A. Sethian. Level Set Methods and Fast Marching Methods: *Evolving interfaces in computational Geometry, Fluid Mechanics, Computer Vision, and Materials Sciences*, 2002

[22] R. P. Beyer, R. J. Leveque. Analysis of a one-dimensional model for the immersed boundary method. SIAM Journal of Numerical Analysis, 29(2): 332–364, 1992.

[23] N. Badshah. Fast iterative methods for variational models in image segmentation. PhD thesis, 2009.

[24] X. F. Wang, D. S. Huang and H. Xu. An efficient local Chan-Vese

model for image segmentation. Pattern Recognition, 43: 603–618, 2010.

[25] H. Ali, N. Badshah, K. Chen, G. A. Khan and N. Zikria. Multiphase segmentation based on new signed pressure force functions and one level set function. Turkish Journal of Electrical Engineering & Computer Sciences, 1: 1-30, 2017.

[26] H. Ali, N. Badshah, K. Chen, G. A. Khan. A Variational Model with Hybrid Images Data Fitting Energies for Segmentation of Images with Intensity Inhomogeneity. Pattern Recognition, http://dx.doi.org/10.1016/j.patcog.2015.08.022.

[27] N. Badshah and K. Chen. Image Selective Segmentation under Geometrical Constraints Using an Active Contour Approach. Math. Comp., 7: 759-778, 2010.

[28] J. Bigun, G. H. Grandlund. Optimal orientation detection of linear symmetry, in: Proceedings of the IEEE First International Conference on Computer Vision (ICCV), 433-438, 1987.

[29] J. Bigun, G. H. Grandlund, J. Wiklund. Multi dimensional orientation estimation with applications to texture analysis and optical flow,IEEE Trans. Pattern Anal. Mach. Intell, 13(8): 775-790, 1991.

[30] T. Brox, J. Weickert, B. Burgeth, P. Mrázek. Non linear structure tensors, Image Vis. Comput., 24(1): 41-55, 2006.

[31] K. Chen. Matrix Preconditioning Techniques and Applications. Cambridge University Press, first edition, 2005.

[32] C. Gout, C. L. Guyader and L. A. Vese. Segmentation under geometrical conditions with geodesic active contour and interpolation using level set method. Numerical Algorithms, 39: 155–173, 2005.

[33] C. L. Guyader and C. Gout. Geodesic active contour under geometrical conditions theory and 3D applications. Numerical Algorithms, 48: 105–133, 2008.

[34] M. Jeon, M. Alexander, W. Pedrycz, and N. Pizzi. Unsupervised hierarchical image segmentation with level set and additive operator splitting, 26(10): 1461–1469, 2005.

[35] M. Kass, A. Witkin and D. Terzopoulos. Active Contours Models. International Jornal of Computer Vision, 321–331, 1988.

[36] C. Li, C. Xu, C. Gui and M. D. Fox Level set evolution without reinitialization: a new variational formulation. IEEE Computer Society Conference on Computer Vision and Pattern Recognition,1: 430436, 2005.

[37] C. Li, R. Huang, Z. Ding, J. C. Gatenby, D. N. Metaxas, J. C. Gore. A level set method for image segmentation in the presence of intensity inhomogeneities with application to MRI, Ieee Transactions on Image Processing, 20(7): 2007–2016, 2011.

[38] C. Li, C. Y. Kao, C. John, Gore and Z. Ding. Implicit Active Contours Driven by Local Binary Fitting Energy. IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pp. 1-7.

[39] T. Lu, P. Neittaanmaki, and X. C. Tai. A parallel splitting up method and its application to Navier-Stokes equations. Appl. Math. Lett., 4(2): 25–29, 1991.

[40] S. Osher and J. A. Sethian. Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations. J. Comput. Phys., 79(1): 12–49, 1988.

[41] L. A. Vese and T. F. Chan. A multiphase level set framework for image segmentation using the Mumford and Shah model, Int. J. Computer Vision, 50(3): 271–293, 2002.

[42] L. A. Vese. Multiphase object detection and image segmentation. Technical report, UCLA Dept. of Math., CAM 02–36, 2002.

[43] B. Wang, X. Gao, D. Tao and X. Li. A Unified Tensor Level Set for Image Segmentation. Ieee Transection on Systems, Man, and Cybernaticspart B: Cybernatics, 40(3): 857–867, 2010.

[44] B. Wang, X. Gao, D. Tao and X. Li. A Nonlinear Adaptive Level Set for Image Segmentation. Ieee Transaction on Cybernatics, 44(3): 418–428, 2014.

[45] B. Wang, X. Gao, D. Tao and X. Li. A relay level set method for automatic image segmentation. Ieee Transection on System, Man, and CyberneticsB, 41(2), 518–525, 2011.

[46] K. Zhang, L. Zhang, H. Song and W. Zhou. Active contours with selective local or global segmentation: A new formulation and level set method. Image and Vision Computing, 28, 668–676, 2010.